STATISTICS I - 2nd Year Management Science BSc - 1st semester – **04/01/2016**

Normal Season Exam – Theoretical Part V1

(theoretical part duration – 30 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (70 points). This answer sheet will be collected 30 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name:______________________________Section:_____Number:____

Each of the following 4 groups of multiple-choice questions is worth 10 points. Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade obtained in each of the 5 groups varies between a minimum of zero and a maximum of 10 points.

Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A, B, C be events of a sample space Ω.

	T	F
If A, B are mutually exclusive events and $P(B) > 0$, then $P(A\|B) = P(B)$.		
$P[A \cup (A' \cap B)] = P(A \cup B)$		
If $B \subset C$, then $P\left(C \cap B'\right) = P(C) - P(B \cap C)$		
Let events A_1, A_2 and A_3 be such that $P(A_1) = 0.4, P(A_2) = 0.2, P(A_3) = 0.3$ and $P(A_i \cap A_j) = 0$ $i, j = 1,2,3$. Then A_1, A_2 and A_3 are a partition of sample space S.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
If X is a discrete random variable, then $F_X(x) \geq P(X < x)$ for any $x \in \mathbb{R}$		
Let $Y = \varphi(X)$ be a function of X. If X is a continuous random variable, then Y can be a discrete random variable.		
If X is a continuous random variable the expected value of X does not always exist.		
If X is discrete, for any $a, b \in \mathbb{R}, a < b$, $P(a \leq X \leq b) = F_X(b) - F_X(a - 0)$.		

3. Let (X, Y)be a two-dimensional random variable.

	T	F
If (X, Y) is a continuous two-dimensional random variable and $E[X\|Y] = E[X]$, then $X, Y$are independent random variables		
If $Cov(X, Y) = 0$, then we can say that X and Y are independent random variables		
If (X, Y) are independent random variables and $M_X(t), M_Y(t)$their moment generating functions, then $M_{X+Y}(t) = M_X(t) + M_Y(t)$		
If X and Y are independent random variables with a common distribution, then $Var(2X - Y) = 5\,Var(X)$		

4.

	T	F
Let $X_1 \sim Po(\lambda_1)$ e $X_2 \sim Po(\lambda_2)$ $\lambda_1 \neq \lambda_2$, be independent random variables, then the random variable $W = X_1 + X_2$ follows a Poisson distribution with mean $\lambda_1 + \lambda_2$.		
Let $(X_1, X_2, \dots, X_n)$ be a random sample of size *n>2* selected from a population X with finite mean and $\bar{X}_1$ and $\bar{X}_2$ be sample means from samples with dimension n of population X. Then $\bar{X}_1, \bar{X}_2$ have a common distribution.		
Let $(X_1, X_2, \dots, X_n)$ be a random sample of size *n>2* selected from a population X. Then, $P(X_1 > x, X_2 > x) = [1 - F_X(x)]^2$ for any value of $x \in \mathbb{R}$		
If random variable $X \sim B(10, 0.1)$, then the distribution of X can be well approximated by a $Po(1)$ distribution,		

The following questions worth 150 points each and should be answered in the space provided. The answers should be duly formalized and justified.

5. Let X be a random variable such that $P(X=1) + P(X=-1) = 1$. Show that $E[X^2] = 1$

6. Se $X \sim Po(\lambda)$, **demonstre** que a sua média é igual a $\frac{P(X=1)}{P(X=0)}$ e a sua variância é igual ao dobro de $\frac{P(X=2)}{P(X=1)}$.

LISBOA SCHOOL OF ECONOMICS & MANAGEMENT — ULISBOA UNIVERSIDADE DE LISBOA

STATISTICS I - 2nd Year Management Science BSc - 1st semester **– 04/01/2016**

Normal Season Exam – Theoretical Part V1

(theoretical part duration – 30 minutes)

This exam consists of two parts. This is Part 1 - Theoretical (70 points). This answer sheet will be collected 15 minutes after the beginning of the examination. During the duration of the exam, no clarifications will be provided. **GOOD LUCK!**

Name:____Section:_____Number:____

Each of the following 4 groups of multiple-choice questions is worth 10 points. Each question answered correctly is worth 2.5 points; each wrong answer is worth -2.5 points. The grade obtained in each of the 5 groups varies between a minimum of zero and a maximum of 10 points.

Indicate whether the following statements are true (**T**) or false (**F**) by ticking the corresponding box with a cross(X):

1. Let A, B, C be events of a sample space Ω.

	T	F
If A, B are independent events and $P(B) > 0$, then $P(A\|B) = P(B)$.		
$P(A \cap B') = P(A) - P(A \cap B)$		
If $C \subset B$, then $P(C \cap B') = 0$		
Let events A_1, A_2 and A_3 be such that $P(A_1) = 0.6, P(A_2) = 0.2, P(A_3) = 0.3$. Then A_1, A_2 and A_3 are a partition of sample space S.		

2. Let X be a random variable with cumulative distribution function $F_X(x)$.

	T	F
If X is a continuous random variable, then $F_X(x) \geq P(X < x)$ for any $x \in \mathbb{R}$		
Let $Y = \varphi(X)$ be a function of X. If X is a mixed random variable, then Y can be a continuous random variable.		
If X is a discrete random variable the expected value of X exists always.		
If X is discrete, for any $a, b \in \mathbb{R}, a < b$, $P(a < X < b) = F_X(b - 0) - F_X(a)$.		

3. Let (X, Y)be a two-dimensional random variable.

	T	F
If (X, Y) is a discrete two-dimensional random variable and $E[X\|Y = y_1] \neq E[X\|Y = y_2]$ $(y_i \in D_Y)$ then $X, Y$are dependent random variables		
If $Cov(X, Y) = 0$, then we can say that X and Y are dependent random variables		
If (X, Y) are independent random variables and $M_X(t), M_Y(t)$their moment generating functions, then $M_{X+Y}(t) = M_X(t) * M_Y(t)$		
If X and Y are independent random variables with a common distribution, then $Var(X - 2Y) = 5\, Var(X)$		

4.

	T	F
Let $Y_1 \sim Bi(n, \theta_1)$ and $Y_2 \sim Bi(n, \theta_2)$ $\theta_1 \neq \theta_2$, be independent random variables, then the random variable $V = Y_1 + Y_2 \sim Bi(2n, \theta_1 + \theta_2)$		
Let $(X_1, X_2, \ldots, X_n)$ be a random sample of size *n>2* selected from a population X and $\bar{X}_1$ and $\bar{X}_2$ be sample means from samples with dimension n of population X. Then $\bar{X}_1$ and $\bar{X}_2$ have the same distribution.		
Let $(X_1, X_2, \ldots, X_n)$ be a random sample of size *n>2* selected from a population X with unknown parameters..Then, the larger the sample size, *n*, the larger $E[\bar{X}]$		
If $X \sim B(1000, 0.8)$, then the distribution of X can be well approximated by a $Po(800)$ distribution,		

The following questions worth 150 points each and should be answered in the space provided. The answers should be duly formalized and justified.

5. Let X be a random variable such that $P(X = 1) + P(X = -1) = 1$. Show that $E[X^2] = 1$

6. Se $X \sim Po(\lambda)$, **demonstre** que a sua média é igual a $\frac{P(X=1)}{P(X=0)}$ e a sua variância é igual ao dobro de $\frac{P(X=2)}{P(X=1)}$.

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STATISTICS I - 2nd Year Management Science BSc - 1st semester – **04/01/2016**
Normal Season Exam – Practical Part

(practical part duration – 90 minutes)

This is Part 2: 130 points. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the space provided.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____ Nº:____________

Don't write here

1a.(10)	2a.(20)	3a.(10)	3c.(15)	4a.(15)	T:
1b.(15)	2b.(15)	3b.(15)		4b.(15)	P:

1. To forecast the daily occurrence of rain (event **R**), three levels of atmospheric pressure (high **H**, medium **M** and low **L**) are considered. The probability of each of the three atmospheric pressure levels are respectively $P(L) = 0.3, P(M) = 0.4, P(H) = 0.3$. Furthermore it is known that the atmospheric pressure is low in 60% of the rainy days and the probability of rain is 20%.

a) In a week, 7 days, compute the probability that atmospheric pressure is high in less than 2 days (signal with an X the right answer,)

(i) 0.3177 ☐ (ii) 0.3294☐ (iii) 0.2471 ☐ (iv) 0.6471 ☐

b) In a day with low atmospheric pressure what is the probability of rain?

2. The daily quantities (tons) of raw materials X and Y embodied in some product are random variables and the function given by:

$$f_{X,Y}(x,y) = k \qquad (0 < x < 2\,;\; 1 < y < 3)$$

a) Find the value of k such that $f_{X,Y}(x,y)$ is the joint probability density function of the random variable (X,Y). Are the variables independent?

b) Find the average quantity of raw material X embodied in the product in a day when 2 tons of raw material Y was used.

3. The number of complaints received in a call center every 10 minutes follows a Poisson distribution with mean 3.

a) The probability that more than 7 calls are received in half an hour is:

(i) 0.9089 ☐ (ii) 0.8829 ☐ (iii) 0.7932 ☐ (iv) 0.6761 ☐

b) Determine the probability that it takes more than 1 hour for the call center to receive 10 calls.

c) If a random sample of 5 periods of 10 minutes were randomly selected find the probability that the maximum number of complaints is lower than 4.

4. A quality standard requires that the weight in grams of a certain type of grain contained in a package differs from the mean weight, in **absolute value**, by less than 5 grams. The weight of cereals contained in a package is well represented by a normal distribution with mean 350 grams and variance 25 grams.

a) Compute the probability that a package does not meet the standard.

b) A random sample of 10 packages were selected. Determine the maximum value of the sample mean with a probability of 90%.

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STATISTICS I - 2nd Year Management Science BSc - 1st semester – **04/01/2016**
Normal Season Exam – Practical Part

(practical part duration – 90 minutes)

This is Part 2: 12 marks. The answers to the multiple-choice questions should be given by signalling with an **X** the corresponding square. The other questions should be answered in the space provided.

Attention: For each of the multiple-choice questions, each correct answer is worth 10 points, each wrong answer is worth -2.5 points.

Name:____
____Nº:____________

Don't write here

1a.(10)	2a.(20)	3a.(10)	3c.(10)	4a.(15)	T:
1b.(15)	2b.(15)	3b.(20)		4b.(15)	P:
_______	________	________	________	________	________

1. To forecast the daily occurrence of rain (event **R**), three levels of atmospheric pressure (high **H**, medium **M** and low **L**) are considered. The probability of each of the three atmospheric pressure levels are respectively $P(L) = 0.3, P(M) = 0.4, P(H) = 0.3$. Furthermore it is known that the atmospheric pressure is high in 15% of the rainy days and the probability of rain is 20%.

a) In a week, 7 days, compute the probability that atmospheric pressure is low in less than 3 days (signal with an X the right answer,)

(i) 0.3177 ☐ (ii) 0.8740 ☐ (iii) 0.2269 ☐ (iv) 0.6471 ☐

b) In a day with high atmospheric pressure what is the probability of rain?

2. The daily quantities (tons) of raw materials X and Y embodied in some product are random variables and the function given by:

$$f_{X,Y}(x,y) = k \qquad (0 < x < 2\,;\; 1 < y < 3)$$

a) Find the value of k such that $f_{X,Y}(x,y)$ is the joint probability density function of the random variable (X,Y). Are the variables independent?

b) Find the average quantity of raw material Y embodied in the product in a day when 1 ton of raw material X was used.

3. The number of complaints received in a call center every 10 minutes has a Poissondistribution with mean 3.

a) The probability that more than 10 calls are received in half an hour is:

(i) 0.2940 ☐ (ii) 0.8682 ☐ (iii) 0.4126 ☐ (iv) 0.8814 ☐

b) Determine the probability that it takes more than 1 hour for the call center to receive 10 calls.

c) If a random sample of 5 periods of 10 minutes were randomly selected find the probability that the maximum number of complaints is lower than 4.

4. A quality standard requires that the weight in grams of a certain type of grain contained in a package differs from the mean weight, in **absolute value**, by less than 5 grams. The weight of cereals contained in a package is well represented by a normal distribution with mean 350 grams and variance 25 grams.

a) Compute the probability that a package does not meet the standard.

b) A random sample of 10 packages were selected. Determine the maximum value of the sample mean with a probability of 90%.